Reg. No. :

Question Paper Code : X 60767

B.E./B.Tech. DEGREE EXAMINATIONS, NOV./DEC. 2020

Second/Third Semester

Civil Engineering

MA 2211/10177 MA 301/MA 1201 A/080100008/080210001/ MA 31

– TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all Branches)

(Regulations 2008/2010)

(Also Common to PTMA2211 for B.E.(Part-Time) Second Semester-All Branches - Regulations 2009)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions.

PART – A **(10×2=20 Marks)**

1. Find the value of a_0 in the Fourier series expansion of $f(x) = e^x$ in $(0, 2\pi)$.

2. Find the half range sine series expansion of $f(x) = 1$ in $(0, 2)$.

3. State Fourier integral theorem.

4. Find the Fourier sine transform of $f(x) = e^{-x/2}$.

5. Form the PDE from $(x - a)^2 + (y - b)^2 + z^2 = r^2$.

6. Find the complete integral of $p + q = pq$.

7. What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation with respect to the time ?

8. Write down the partial differential equation that represents steady state heat flow in two dimensions and name the variables involved.

9. Find the Z-transform of a^n.

10. Solve $y_{n+1} - 2\,y_n = 0$, given that $y(0) = 2$.

PART – B (5×16=80 Marks)

11. a) i) Find the Fourier series of x^2 in $(-\pi, \pi)$ and hence deduce that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\frac{\pi^4}{90}$. **(8)**

ii) Obtain the Fourier cosine series of $f(x)=\begin{cases} kx, & 0<x<\frac{1}{2} \\ k(l-x), & \frac{1}{2}<x<l \end{cases}$. **(8)**

(OR)

b) i) Find the complex form of Fourier series of cos ax in $(-\pi, \pi)$, where "a" is not an integer. **(8)**

ii) Obtain the Fourier cosine series of $(x-1)^2$, $0 < x < 1$ and hence show that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots=\frac{\pi^2}{6}$. **(8)**

12. a) i) Find the Fourier transform of $f(x)=\begin{Bmatrix} 1-|x| & \text{if} & |x|<1 \\ 0, & \text{if} & |x|>1 \end{Bmatrix}$ and hence evaluate $\int_0^\infty \frac{\sin^4 t}{t^4}\,dt$. **(8)**

ii) Find the Fourier transform of $f(x)=\begin{Bmatrix} a^2-x^2, & |x|\le a \\ 0, & |x|>a>0 \end{Bmatrix}$. Hence deduce that $\int_0^\infty \frac{\sin t - t\cos t}{t^3}\,dt=\frac{\pi}{4}$. **(8)**

(OR)

b) i) Find the Fourier cosine and sine transforms of $f(x) = e^{-ax}$, $a > 0$ and hence deduce the inversion formula. **(8)**

ii) Find the Fourier cosine transform of $e^{-a^2x^2}$, $a > 0$. Hence show that the function $e^{-x^2/2}$ is self-reciprocal. **(8)**

13. a) i) From the PDE by eliminating the arbitrary functions f_1, f_2 from the relation $Z = xf_1(x + t) + f_2(x + t)$. **(8)**

ii) Solve $\left(\frac{p}{2}+x\right)^2+\left(\frac{q}{2}+y\right)^2=1$. **(8)**

(OR)

b) i) Solve $x^2p + y^2q = z(x + y)$. **(8)**

ii) Solve $(r + s - 6t) = y \cos x$. **(8)**

14. a) A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by $y(x, 0) = K(lx - x^2)$. It is released from rest from this positions. Find the expression for the displacement at any time 't'. **(16)**

(OR)

b) Find the solution to the equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ that satisfies the conditions,

$u(0, t) = 0$, $u(l, t) = 0$ for $t > 0$ and $u(x, 0) = \begin{cases} x, & 0 \le x \le l/2 \\ l - x, & l/2 < x < l \end{cases}$. **(16)**

15. a) i) Find $Z[n(n-1)(n-2)]$. **(8)**

ii) Using Convolution theorem, find the inverse Z-transform of $\frac{8z^2}{(2z-1)(4z-1)}$. **(8)**

(OR)

b) i) Solve the difference equation $y(k + 2) + y(k) = 1$, $y(0) = y(1) = 0$, using Z-transform. **(8)**

ii) Solve $y_{n+2} + y_n = 2^n.n$, using Z-transform. **(8)**
